Zonal LP Dispatch

Introduction

This document describes Aurora's mathematical formulation of the zonal dispatch problem. A linear program (LP) is used to determine the optimal resource generation pattern for a given time period based on the resource characteristics, network topology, and demand levels. The problem is solved for each hour or sub-hour period after the commitment status has been determined for commitment units; this is in direct contrast to commitment optimization which simultaneously solves both. The LP guarantees to find the minimum cost solution given the zonal demand, transmission flow limits, resource minimum and maximum limits, resource costs, and all other user-defined constraints. The decision variables include the dispatch in MW for each resource as well as the transmission flow between each pair of zones. The objective function is the total wheeling cost of transmission flows plus the total production cost of generating the resources as defined by their dispatch costs. The solution (found using a third-party solver) gives the generation for each resource, the transmission flows, and the price of power for each zone.

Mathematical Framework

Notation

Let $R_1, R_2, ..., R_n$ be the set of resource segments in the system, and let $Z_1, Z_2, ..., Z_m$ be the set of zones. Note that each resource must belong to exactly one zone. Each resource is made up of one or more segments, each of which has its own dispatch cost and capability. Resources defined by the user to be Must Run or Non Cycling (commitment) units have a minimum segment. The minimum segment is treated differently from the other segments as shown in the formulation below. The bounds on the variable representing a minimum segment go from 0 to 1, whereas the bounds on the dispatchable (non-minimum) segments range from 0 to the segment capability. The status of the minimum segments is known before solving this LP; these values are set based on the results of the commitment algorithm. Note that in the mathematical notation below (with the exception of subscripts), all lower-case letters represent decision variables and all upper-case letters represent data input by the user or derived beforehand by the model.

Let M be the set of all minimum resource segments, and let H be the set of all dispatchable resource segments. Define the following:

r_i represents the dispatch for resource segment R_i (decision variables)
t_{ij} = transmission from Z_i to Z_j (decision variables)
U_i = capability of resource segment R_i
C_i = dispatch cost of resource segment R_i
T_{ij} = transmission capacity from Z_i to Z_j
L_{ij} = loss factor for transmission from Z_i to Z_j (e.g. if 1% of flow from Z_i to Z_j is lost, then $L_{ij} = 0.99$)
W_{ij} = wheeling charge for transmission from Z_i to Z_j
D_j = demand for zone Z_j

Formulation

The model creates the following objective function to represent the total cost of the system. Total cost is the sum of total resource costs and total wheeling costs:

$$\sum_{R_i \in M} r_i U_i C_i + \sum_{R_i \in H} r_i C_i + \sum_{i=1}^{m} \sum_{j=1}^{m} t_{ij} W_{ij} \tag{1}$$

Bounds on the resource variables ensure that minimum and maximum generation requirements are obeyed. More specifically, for all $i \in \{1, 2, ..., n\}$,

If $R_i \in H$, then

$$0 \leq r_i \leq U_i \tag{2}$$

If $R_i \in M$ and R_i is a must run unit, then

$$r_i = 1 \tag{3}$$

If $R_i \in M$ and R_i is a commitment unit, then if the unit is committed, the dispatch cost is decreased by the user-defined min gen back down penalty, and if the unit is not committed, the dispatch cost is increased by the non-commitment penalty. This ensures that in nearly all cases the variable result will be either 0 or 1. The bounds then are set as

$$0 \leq r_i \leq 1 \tag{4}$$

Transmission constraints require that transmission flows between zones are within capacity. For all $i, j \in \{1, 2, ..., m\}$ (with $T_{ij} = 0$ for $i = j$):

$$0 \leq t_{ij} \leq T_{ij} \tag{5}$$

Tiered wheeling rates may also be specified on a given zone-to-zone link, and in that case constraints are added to properly model those tiered costs.

Demand constraints require that total supply equals total demand in each zone. For all $j \in \{1, 2, ..., m\}$:

$$D_j = \sum_{R_i \in Z_j \cap M} r_i U_i + \sum_{R_i \in Z_j \cap H} r_i + \sum_{i=1}^{m} (L_{ij} t_{ij} - t_{ji}) \tag{6}$$

The shadow prices (i.e. dual variable values) on these demand constraints are reported as the zonal prices.

To summarize, the model formulates the following LP and finds the vectors $\boldsymbol{r}$ and $\boldsymbol{t}$ of resource dispatch and transmission flows between zones, respectively:

$$\begin{array}{lll}
\text{minimize} & \sum_{R_i \in M} r_i U_i C_i + \sum_{R_i \in H} r_i C_i + \sum_{i=1}^{m} \sum_{j=1}^{m} t_{ij} W_{ij} & \\
\text{subject to} & 0 \leq r_i \leq U_i & R_i \in H \\
 & r_i = 1 & R_i \in M \text{ is a must run unit} \\
 & 0 \leq r_i \leq 1 & R_i \in M \text{ is a commitment unit} \\
 & 0 \leq t_{ij} \leq T_{ij} & \text{for all } i, j \\
 & D_j = \sum_{R_i \in Z_j \cap M} r_i U_i + \sum_{R_i \in Z_j \cap H} r_i + \sum_{i=1}^{m} (L_{ij} t_{ij} - t_{ji}) & \text{for all } j
\end{array}$$

Additional Constraints

In addition to the constraints outlined above, the user may also impose other constraints on the model. These may include any of the following:

Ramp down constraint: This ensures that a unit cannot decrease its output by more than a specified ramp down amount (input as a percent). If a unit ran at Y MW in hour h (not counting the output of the minimum segment), then this will require that in hour $h+1$ the dispatchable segments of the unit will run at least at $X = Y - (\mathit{rampDownRate} \times \mathit{capacity})$ MW. Let S be the set of all dispatchable segments of a particular resource.

$$\sum_{R_i \in S} r_i \geq X \tag{7}$$

Multiple fuel constraint: For a resource that has $k > 1$ fuel sources defined, this limits the resource segment R_i to a utilization of its fuels such that its total MW output from all fuels is exactly equal to the MW output of the resource segment. Let r_{i_j} denote the output of resource segment R_i generated by utilization of fuel j. The model will commit on a given fuel, and then the constraint will hold for all dispatchable resource segments. In this case, the model will also account for varying fuel costs in the objective cost function. The constraint requires that for $R_i \in H$ the following must hold:

$$\sum_{j=1}^{k} r_{i_j} = r_i \tag{8}$$

Hourly fuel constraint: This constrains all resource segments that utilize the specified fuel, directly or indirectly, to a cumulative limit of X MMBtu of fuel per hour. Let S be the set of all resource segments that use the fuel, and call it fuel j. Let r_{i_j} denote the output of resource segment R_i generated by utilization of fuel j. Let Q_i denote the heat rate of resource segment R_i in Btu/kWh.

$$\sum_{R_i \in S \cap M} \frac{Q_i r_{i_j} U_i}{1000} + \sum_{R_i \in S \cap H} \frac{Q_i r_{i_j}}{1000} \leq X \tag{9}$$

Operating reserve constraint: This ensures that enough resource capacity is set aside to meet a zonal operating reserve requirement. Let X MW be the requirement for a given zone or pool after the hydro reserve contribution (which is known beforehand) is netted out. Let S be the set of resource segments that can contribute to the operating reserve constraint.

$$\sum_{R_i \in S} \left(U_i - \begin{cases} r_i U_i & R_i \in M \\ r_i & R_i \in H \end{cases} \right) \geq X \tag{10}$$

Spinning reserve constraint: This ensures that enough resource capacity is set aside to meet a spinning reserve requirement. Let X MW be the requirement for a given pool after the hydro reserve contribution (which is known beforehand) is netted out. Let S be the set of all resource segments belonging to resources with minimum segments online or which have the parameter Fast Start For Spin set to True.

$$\sum_{R_i \in S} \left(U_i - \begin{cases} r_i U_i & R_i \in M \\ r_i & R_i \in H \end{cases} \right) \geq X \tag{11}$$

Generation MW constraint: This limits the total generation of a set of p resources to be less than, equal to, or greater than some target value X MW. For all $j \in \{1, 2, ..., p\}$, the j^{th} resource defines a set $R^{(j)}$ of resource segments belonging to it. The user may also add a multiplier α_j on the generation of each of the resources.

$$\sum_{j=1}^{p} \alpha_j \left(\sum_{R_i \in M \cap R^{(j)}} r_i U_i + \sum_{R_i \in H \cap R^{(j)}} r_i \right) \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} X \tag{12}$$

Multilink limits: This places an upper or lower limit X MW on the net flow of a set S of transmission links. Let S be the set of directed zonal links defined as part of the multilink limit.

$$\sum_{t_{ij} \in S} (t_{ij} - t_{ji}) \begin{Bmatrix} \leq \\ \geq \end{Bmatrix} X \tag{13}$$

